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On Optimal Reservation Prices in Auctions

Richard Engelbrecht-Wiggans

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

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Richard Engelbrecht-Wiggans, Associate Professor
Department of Business Administration


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Abstract

The theory of auction design examines how various factors affect the outcome of an auction. Most of the existing literature focuses on how varying the amount of information available to each bidder affects the bid-taker's expected revenue when all other factors remain constant. This paper studies how the bid-taker's expected revenue varies with changes in the auction format when such changes affect the number of bidders. Specifically, we examine how varying the reservation price or screening level affects the bid-taker's expected revenue though its effect on the number of bidders. For two simple examples, the losses associated with a reduced number of bidders outweighs any benefits that non-trivial reservation prices might have had in models with an exogenously set number of bidders.



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Introduction

The bid-taker's expected revenue from an auction depends on how the bidders will bid, which in turn, depends on a variety of factors. The auction rules announced by the bid-taker -- rules specifying how the bidding will be conducted, and how the bidding will determine who wins what and who pays whom how much -- typically affect the bidding. The number of bidders, and the type and amount of information held by each bidder about the underlying true state of nature may also affect the bidding. In addition, the bidders' utilities for different auction outcomes as a function of the true state, and the distribution of the true state itself often affects the bidding.

The theory of auction design attempts to understand, describe, and predict how the bid-taker's expected revenue depends on the factors that affect the bidding. To do so, the theory must also model -- or make appropriate assumptions about -- how the bids made -- or certain aggregated characteristics or consequences of the bids made -- depend on what the bidders know about the auction rules, the true state of nature, and their own preferences. Here the theory draws on concepts ranging from dominant strategies to a variety of different types of equilibria. As a result, the predicted expected revenue varies not only with the factors that affect how bidders bid, but also with the assumptions on how these factors affect the bidding.

The early literature on auction design mainly compares the expected outcome in one specific auction with that in another specific auction. For example, Vickrey (1961) considers a model of auctions in which each bidder knows his own value for the single object being sold, and

that other bidders' values and information are independent of, and identically distributed with, his own. He examines the Nash equilibria of the first price and the second price sealed bid versions of this independent private values model, and discovers that at equilibrium, both auctions yield the same expected revenue to the bid-taker. More recently, Milgrom and Weber (1982) formulated a much more general model of auctions of a single object to risk neutral bidders, and among other results, compare the expected revenue of the bid-taker at the corresponding Nash equilibrium for three specific types of auctions. Roughly speaking, the more (affiliated) information each bidder has, the higher the bid-taker's expected revenue.

The later literature on auction design tends to suppress the details of the auction rules and of how the various factors affect how bidders bid. Instead, it simply places restrictions directly on how the outcome of the auction may depend on the bidders' information, number, and preferences. These restrictions implicitly define families of auctions; the literature examines how the outcome -- sometimes focusing exclusively on the bid-taker's expected revenue -- depends on the various factors that might affect the outcome. For example, Myerson (1981) considers all mechanisms satisfying two conditions within the independent private values model. Roughly speaking, one condition requires that each bidder have an alternative in effect identical to withdrawing from the auction entirely, while the other condition requires that no bidder have an incentive to lie to himself about the information that only he observes. As a result, the bid-taker's expected revenue depends only on how the final award of the

object relates to the bidders' values. In particular, any mechanism satisfying Myerson's conditions (in an independent private values model) that always awards the object to the bidder valuing it most highly will generate the same expected revenue for the bid-taker -- a generalization of Vickrey's result.

In addition, Myerson examines how the bid-taker's expected revenue varies with the allocation rule. In particular, he discovers that to maximize the bid-taker's expected revenue, the object should be awarded to the bidder who values it most highly if and only if this highest value exceeds some critically chosen reservation or screening level; otherwise, the bid-taker keeps the object. This optimal allocation rule gives a positive probability that the bid-taker retains the object even though some bidder has offered to pay more than the bid-taker's value for the object.

Then why do real world auctions advertise "all items will be sold without reserve?" Why do many states outlaw shills -- agents of the bid-taker who bid like other bidders and have the same effect as an unknown reservation price? Are these practices misguided, or do the traditional models of auctions and competitive bidding miss something?

Up to now, the auction design literature focused on models that exogenously specify the number of bidders and what private information each bidder observes, and studies the affect on the bid-taker's expected revenue of varying amounts of this (and any private information of the bid-taker) being revealed to everyone. In contrast, we shall study how the bid-taker's expected revenue varies with the auction

format when such changes affect the number of bidders or the amount of private information acquired by each. Specifically, we will examine the affect of the screening level on the bid-taker's expected revenue when the screening level itself affects the number of bidders (possibly through its affect on the bidders' expected profits). In at least some cases, the loss of bidders from using a non-trivial reservation price hurts the bid-taker's expected revenue more than the non-trivial reservation price's beneficial effect on the expected revenue from a fixed number of bidders.

An Example:

This section presents an example illustrating the types of questions that might be asked, the methods that might be used, and the new insights that might be gained studying models of auctions in which the number of bidders, or the amount of information acquired by the bidders, might vary with the auction rules. In particular, we will vary the reservation price -- or more accurately, the screening level -- in a specific independent private values model in which the number of bidders might decrease as the screening level increases. The maximum possible expected revenue for the bid-taker results from increasing the screening level from zero until either the number of bidders drops, or until the optimal screening level for a fixed number of bidders model is reached -- whichever happens first. In general, this screening level will be less than that derived by Myerson for models with a fixed number of bidders.

Specifically, consider the following independent private values model: each of the n risk neutral bidders knows his own value v for the object being sold and that the bidders' values are independent draws from the known cumulative probability distribution $F(v)$. (For the moment, think of n as a fixed, positive integer.) The object will be awarded to the bidder who values it most highly, so long as this value exceeds the known screening level r ; otherwise, the bid-taker destroys the object. (Without any loss of generality, restrict r to lie in the support of F .) Then, by performing the calculations in the context of a second price sealed bid auction, but appealing to the equivalence results of Myerson or of Engelbrecht-Wiggans (1985a,b; 1986) for auctions with independent information, the bid-taker has an expected revenue $R(r,n)$ of $nr(1-F(r))(F(r))^{n-1} + n(n-1) \int_{v \geq r} v(1-F(v))(f(v))^{n-2} dF(v)$. Note that if the appropriate derivatives exist and if the r that maximizes $R(r,n)$ lies strictly within the support of F , then the optimal r must satisfy the first order condition $\frac{d}{dr} R(r,n) = 0$; this condition reduces to $1-F(r) = rf(r)$, where $f(v)$ denotes the probability density function $\frac{d}{dv} F(v)$.

For our example, we assume the bidders' values to be uniformly distributed on the unit interval. This yields an expected bid-taker revenue of $\frac{n-1}{n+1} + r^n - \frac{2n}{n+1} r^{n+1}$. Note that for this example, 1) $R(r,n)$ has a unique maximum with respect to r at r equals one half, 2) $R(r,n)$ is strictly increasing with respect to n , and 3) $R(0,n+1) - R(1/2,n)$ has a (unique) minimum with respect to n at n equals $\frac{1 - \ln(2)}{\ln(2)}$ (which is less than unity), and has a strictly positive minimum value.

Now, assume that n will not increase, and may decrease, as r increases. Then, in our example, for r greater than one-half, the bid-taker's expected revenue would be increased by reducing r to one-half, both because this would maximize the expected revenue if the number of bidders remained fixed, and because, in addition, the number of bidders may increase, thereby further increasing the bid-taker's expected revenue. Therefore, the optimal r will be at most one-half. However, for any r_1 less than or equal to one-half, $R(r_1, n+1)$ exceeds $R(0, n+1)$ (by the fact that R has a unique maximum at r equal to one-half, and must therefore be an increasing function of r for r less than one-half). But now, the third fact noted above implies that $R(0, n+1)$ exceeds $R(1/2, n)$. Finally, R having a unique maximum at r equal to one-half implies that $R(1/2, n)$ exceeds $R(r_2, n)$ for any r_2 . Taken together, we have that for any r_1 less than or equal to one-half, and any r_2 at all, and any n , $R(r_1, n+1)$ exceeds $R(r_2, n)$. Therefore, a screening level of r maximizes the bid-taker's expected revenue only if it is less than or equal to one-half and if it results in the largest possible number of bidders that would bid under any value of r ; if more than one value of r satisfies these necessary conditions, then the monotonicity of R in r for r less than one-half implies that the optimal r will be the largest r satisfying the necessary conditions.

Note that for a fixed number of bidders in our example, the bidders' expected profit decreases as the screening level increases from zero to one-half and as the number of bidders increases. Therefore, if an individual demands at least some minimal, but strictly

positive, expected profit -- perhaps to simply cover the costs of obtaining the private information or of preparing a bid -- then the number of individuals willing to bid will decrease as the screening level increases from zero to one-half. Thus, the optimal screening level in this example will typically be less than one-half -- the optimal screening level in this model for any fixed number of bidders.

In this example, the number of bidders only changes in discrete steps; this lets us avoid having to model explicitly how changing the reservation price affects the number of bidders. As a result, a non-trivial reservation will still be optimal so long as the number of bidders bidding is the maximum possible (in other words, the number that would bid if the reservation price were zero). However the example does suggest the possibility that in models where the (expected) number of bidders, or the amount of information each acquires, may vary continuously with the reservation price, no non-trivial reservation price will ever be optimal. The following section presents an example illustrating this possibility.

Another Example:

This example illustrates the possible effects of a non-trivial reservation price on the bid-taker's expected revenue when the expected number of bidders varies continuously with the screening level r . To motivate the example, think of a market with many potential bidders, each of which participates in any specific auction with some probability, and that the probability of bidding will be chosen to vary with r so that on average the bidders actually bidding just

recover some fixed minimal costs. Specifically, modify the previous section's example so that the number of bidders bidding in any specific auction may be viewed as the outcome of a Poisson distributed random variable, a random variable whose outcome is independent of the bidders' values for the object, but whose mean λ varies with r so that the bidders' total expected profit divided by the mean number of bidders -- in a sense, the expected profit per bidder -- remains constant.

Again, performing the calculations in terms of a second price sealed bid auction and again appealing to the known equivalence results for auctions with independent values, for fixed n and r the expected bid-taker revenue $R(r, n)$ will be as before, while the winning bidder's expected value for the object $V(r, n)$ will be

$n \int_{v \geq r} v (F(v))^{n-1} dF(v)$. Now, by using a probability $e^{-\lambda} \lambda^n / n!$ of n bidders bidding, and averaging over all possible numbers of bidders, we

get that for any fixed r and λ , the bid-taker's expected revenue

$R^*(r, \lambda)$ equals $\lambda r(1-F(r))e^{\lambda(F(r)-1)} + \lambda^2 \int_{v \geq r} v(1-F(v))e^{\lambda(F(v)-1)} dF(v)$

and the winner's expected value $V^*(r, \lambda)$ equals $\int_{v \geq r} \lambda v e^{\lambda(F(v)-1)} dF(v)$.

We assume that λ varies with r so that for some constant c (independent of λ and r), $V^*(r, \lambda) - R^*(r, \lambda) = c\lambda$. As before, the first order condition for the r that maximizes $R^*(r, \lambda)$ is $1-F(r) = rf(r)$.

For bidders' values distributed uniformly on the unit interval,

$R^*(r, \lambda) = (1 - \frac{2}{\lambda}) + e^{\lambda(r-1)}(1-2r + \frac{2}{\lambda})$, $V^*(r, \lambda) =$

$(1 - \frac{1}{\lambda}) + (\frac{1}{\lambda} - r)e^{-\lambda(r-1)}$, and the expected profit per bidder

$(R^*(r, \lambda) - V^*(r, \lambda))/\lambda = \frac{1}{\lambda^2} [1 - (1 - \lambda(r-1))e^{\lambda(r-1)}]$. Setting this

expected profit per bidder equal to a constant and implicitly differentiating the resulting expression gives

$$\frac{dr}{d\lambda} = \frac{2[1 - (1 - \lambda(r-1)) + \frac{\lambda^2}{2}(r-1)^2]e^{\lambda(r-1)}}{\lambda^3(1-r)e^{\lambda(r-1)}}$$

(note that for algebraic simplicity, we have chosen to view r as varying with λ rather than vice versa). Using the Taylor series expansion $e^{-\lambda(r-1)} = 1 - \lambda(r-1) + \frac{\lambda^2}{2}(r-1)^2 + \delta$ for some strictly positive δ establishes that $\frac{dr}{d\lambda} < 0$ for all λ and r .

Now, look at how the bid-taker's expected revenue varies with r and λ when r and λ are linked by the above expression for $\frac{dr}{d\lambda}$. In particular,

$$\frac{dR^*(r, \lambda)}{d\lambda} = \frac{2}{\lambda^2} + [(r-1)(1-2r + \frac{2}{\lambda}) - \frac{2}{\lambda^2}]e^{\lambda(r-1)} + \lambda(1-2r)e^{\lambda(r-1)} \frac{dr}{d\lambda}$$

which reduces to $\frac{e^{\lambda(r-1)}}{\lambda^2} [\lambda^2 r(1-r)^2 - \delta(6r-4)]$ through an appropriate

use of the previous Taylor series expansion and the previous expression for $\frac{dr}{d\lambda}$. Note that for r less than or equal to two-thirds, the bid-taker's expected revenue increases as λ increases.

As in the previous example, we need only consider screening levels r of at most one-half. In this range of interest, the bid-taker's expected revenue increases with λ . However, we have also established that λ increases as r decreases. So, the bid-taker's expected revenue decreases as r increases. Therefore, to maximize his expected revenue,

the bid-taker should use the trivial screening level $r = 0$, and as a result always award the object to the bidder valuing it most highly.

As in the previous example, the reduction in the bid-takers' expected revenue resulting from any decrease in the expected number of bidders outweighs any possible benefits that a non-trivial reservation price might have had if the number of bidders had been fixed. Whereas certain non-trivial reservation prices still resulted in the maximum possible number of bidders in the previous example, only a trivial reservation price results in the maximum possible mean number of bidders in the second example. However, the possibility remains that in yet another example the number of bidders will vary so slowly (but still continuously) in the screening level that the maximum possible expected revenue for the bid-taker results from a non-trivial reservation price and a corresponding less than maximum possible number of bidders; the examples of this paper only show that trivial reservation prices -- and the corresponding ex post efficient auctions -- maximize the bid-taker's expected revenue in at least some plausible instances.

Summary:

The existing theory of auctions and competitive bidding focuses on models with a fixed number of bidders. This typically results in the bid-taker's expected utility being maximized by an ex post inefficient auction -- more specifically, by an auction with a non-trivial reservation price or screening level. However, these results appear to conflict with certain real world practices.

This paper investigates the possibility that the optimality of non-trivial reservation prices arises from assuming a fixed number of bidders. In particular, we let the number of bidders vary with the screening level. For two straightforward examples, any decrease in the (expected) number of bidders hurts the bid-taker's expected revenue more than any benefits from a non-trivial reservation price. For the one example in which the expected number of bidders varies strictly with the reservation price, the ex post efficient auction -- the auction with a trivial reservation price -- maximizes the bid-taker's expected revenue. This highlights the important, and possibly subtle, ramifications of assuming a fixed number of bidders, and suggests a possible explanation for observed real world practices.

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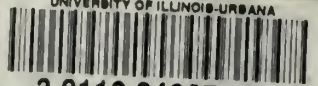
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